Journal of Taibah University for Science xxx (2016) xxx–xxx
www.elsevier.com/locate/jtusci

Weighting order and disorder on complexity measures

José Roberto C. Piqueira [a,b,*]

[a] *Escola Politécnica da Universidade de São Paulo, National Institute of Technology for Complex Systems, Brazil*
[b] *Avenida Prof. Luciano Gualberto, travessa 3, n. 158, 05508-900 São Paulo, SP, Brazil*

Received 13 March 2016; received in revised form 25 April 2016; accepted 16 May 2016

Abstract

The initial ideas regarding measuring complexity appeared in computer science, with the concept of computational algorithms. As a consequence, the equivalence between algorithm complexity and informational entropy was shown. Attempting to connect these abstract formalisms to natural phenomena, described by Thermodynamics, the maximum disorder of a system would correspond to maximum complexity, a fact incoherent with the intuitive ideas of natural complexity. Considering that natural complexity resides in the half path between order and disorder, López-Ruiz, Mancini and Calbet proposed a definition for complexity, which is referred as LMC measure. Shiner, Davison and Landsberg, by slightly changing the definition of LMC, proposed the SDL measure. However, there are some situations where complexity is more associated to order than to disorder and vice-versa. Here, a computational study concerning weighting order and disorder in LMC and SDL measures is presented, by using a binomial probability distribution as reference, showing the qualitative equivalence between them and how the weight changes complexity.

Keywords: Binomial; Complexity; Information; Measure; Probability

1. Introduction

When Allan Turing developed the concept of universal Turing machine (UTM), the computational processes could be measured considering the number of operations that is necessary to the UTM, to perform the algorithm related to them [1].

Considering general computational algorithms, Kolmogorov formalized the concept of computational complexity in a rigorous mathematical way [2], which is very useful for software and hardware designers [3].

The parallel development of Shannon information theory [4] allowed strong improvements in communication technology. Moreover, the concept of informational entropy became popular and it has been applied in several areas of knowledge [5].

Concerning to computational complexity, it was shown that measuring it through an algorithm approach is equivalent to using the concept of informational entropy [3]. Consequently, maximum computational complexity corresponds to an equiprobable process [3] and, extending the idea to Thermodynamics, is related to thermodynamical equilibrium [6].

Consequently, this approach is not convenient to be applied to natural phenomena because, as asserted by

* Tel.: +55 1130915221.
E-mail address: piqueira@lac.usp.br
Peer review under responsibility of Taibah University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jtusci.2016.05.003
1658-3655

Anand and Orlóci [7], and Kaneko and Tsuda [8], there is no complexity in situations presenting zero or maximum entropy.

In order to solve this problem, López-Ruiz, Mancini and Calbet proposed a definition for complexity, the LMC measure [9]. Shiner, Davison and Landsberg, by slightly changing the definition of LMC, proposed the SDL measure [10].

Both measures combine equilibrium (disorder) and disequilibrium (order) parameters resulting in expressions for complexity. Several problems were successfully studied with this approach as, for instance, in Biology [11,12] and Meteorology [13], but with criticism concerning the same weight to order and disorder.

In Neuroscience, for instance, it is claimed that complexity is due to order [14], in Psychiatry to disorder [15]. However, weighting order and disorder according to the situation to be analyzed is simple for LMC or SDL measures. Here, a method of weighting order and disorder is developed, using the support of binomial distribution [16] and showing how weights affect complexity.

In the next section, some theoretical hints are presented, emphasizing how to introduce weights in LMC and SDL complexity measures. Then, a section of computational experiments, with the binomial distribution, shows the numerical effects of weighting complexity measures. A section of conclusions completes the work.

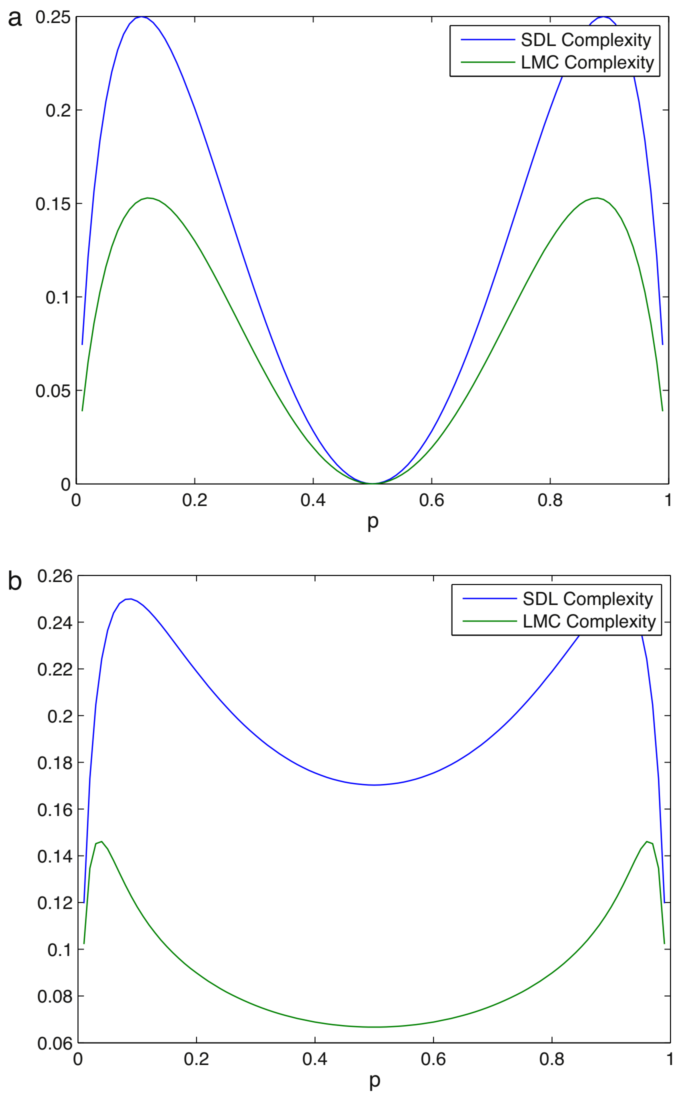

Fig. 1. Complexity measures for $\alpha = 1.0$: (a) $N = 1$ and (b) $N = 10$.

2. Weighting LMC and SDL complexity measures

Both complexity measures, LMC and SDL, are based on the product of two parameters, one related to the measure of disorder or thermodynamical equilibrium (Δ), and the other related to order $(1-\Delta)$, or to thermodynamical disequilibrium (D).

In order to calculated them, a probability distribution p_i for the N possible discrete states of the system must known. With this knowledge, it is possible to calculate the informational entropy, E, in bits per state, by using:

$$E=-\sum_{i=0}^{N} p_i \log_2 p_i, \tag{1}$$

having its maximum value given by: $E_{max}=\log_2 N$ [4].

Consequently, it is possible to measure the system disorder, Δ, measuring the thermodynamic equilibrium [9,10], with:

$$\Delta=\frac{E}{E_{max}}. \tag{2}$$

As stated in [10], combining disorder, Δ and order $(1-\Delta)$, the SDL complexity measure, C_{SDL}, can be defined as:

$$C_{SDL}=\Delta(1-\Delta), \tag{3}$$

meaning that the maximum complexity corresponds to an equal balance between order and disorder.

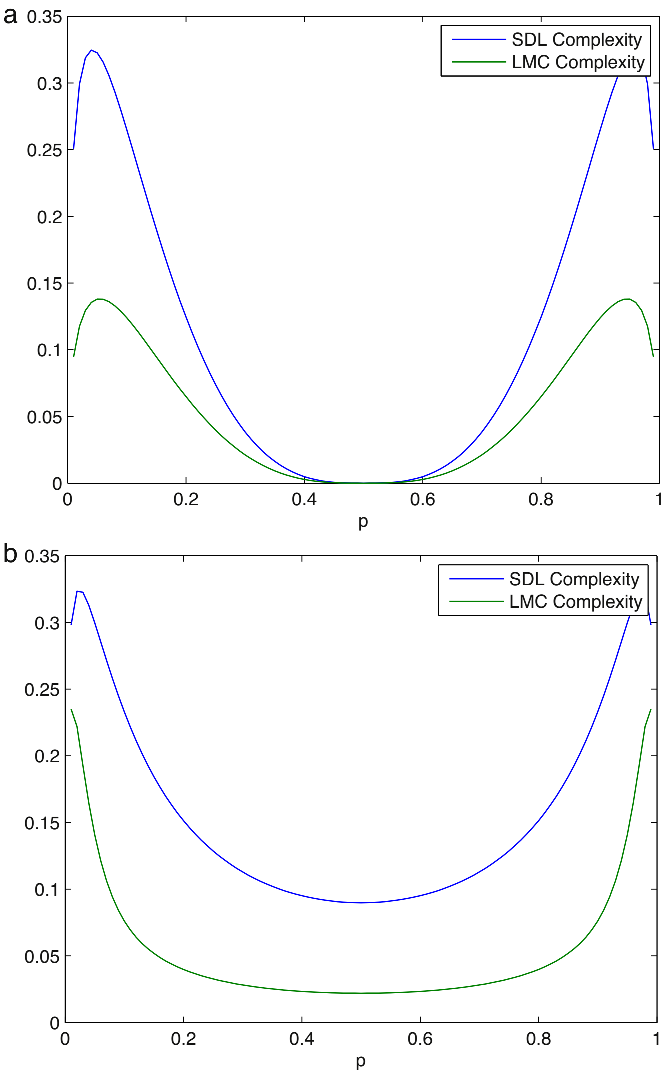

Fig. 2. Complexity measures for $\alpha=0.5$: (a) $N=1$ and (b) $N=10$.

The LMC complexity measure contains a term called disequilibrium D, which was replaced by the order term $(1-\Delta)$ in SDL complexity measure. The disequilibrium D measures the distance between the probability distribution and the equiprobable one, and is defined by:

$$D=\sum_{i=0}^{N}\left(p_i-\frac{1}{N}\right)^2. \tag{4}$$

Then, LMC complexity measure is given by:

$$C_{\mathrm{LMC}}=\Delta D. \tag{5}$$

Here, the expressions of SDL (3) and LMC (5) measures will be modified introducing weight exponents: α for the disorder (equilibrium term) and β for the order (disequilibrium term), following a normalization condition, $\alpha+\beta=2$.

Consequently:

$$\begin{aligned} C_{\mathrm{SDL}} &= \Delta^{\alpha}(1-\Delta)^{2-\alpha},\\ C_{\mathrm{LMC}} &= \Delta^{\alpha} D^{2-\alpha}. \end{aligned} \tag{6}$$

3. Numerical experiments

The numerical experiments to be conducted are based on the binomial probability distribution, composed of N binomial independent trials with individual success

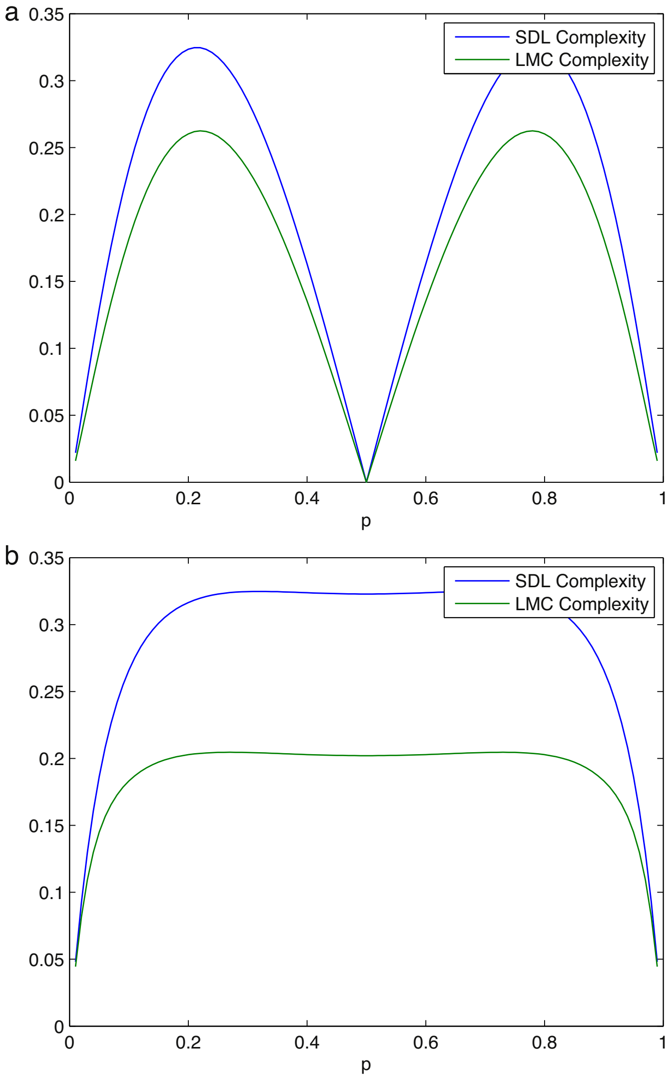

Fig. 3. Complexity measures for $\alpha=1.5$: (a) $N=1$ and (b) $N=10$.

probability p. Consequently, the probability density function is discrete and given by:

$$p(i) = \binom{N}{i} p^i (1-p)^{N-i}, \qquad (7)$$

for $i = 0, 1, \ldots, N$ [17].

The idea is to study how the complexity measures given by Eqs. (6) are affected by varying the exponent α.

If the exponential parameters are equal, $\alpha = \beta = 1$, i.e., attributing equal weights to order and disorder, for $N = 1$ (Fig. 1a) and $N = 10$ (Fig. 1b).

Fig. 1a shows that, when order and disorders have the same weight, LMC and SDL measures depend on the individual success probability in a symmetrical way, related to equiprobability, vanishing under this condition.

When the number of trials increases, as shown in Fig. 1b, symmetry is maintained, but in the equiprobable case, the complexity measures present non-zero minimum values. These results are compatible with the presented in [16].

Considering $\alpha = 0.5$, i.e., considering order (disequilibrium) more important than disorder (equilibrium), the results are shown in Fig. 2a, for $N = 1$, and Fig. 2b, for $N = 10$.

The symmetry related to equiprobability is maintained, however, related to the former case, the maximum possible values for LMC and SDL measures is slightly increased. Besides, as Fig. 2b shows, in the equiprobable

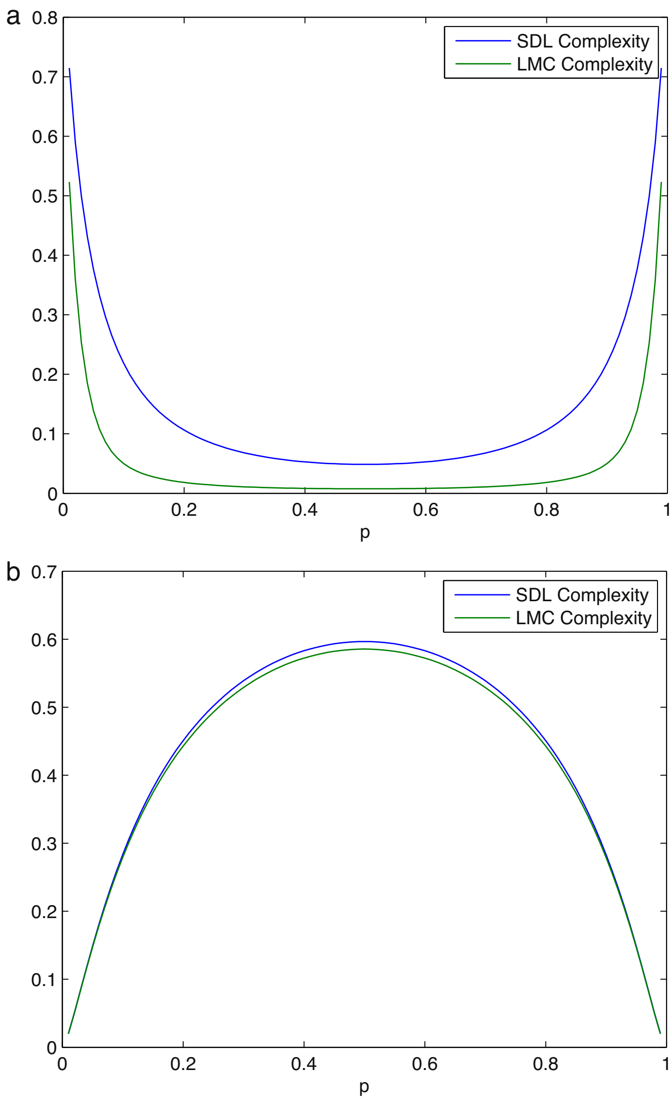

Fig. 4. Complexity measures exaggerating weights ($N = 10$): (a) $\alpha = 0.02$ and (b) $\alpha = 1.98$.

Please cite this article in press as: J.R.C. Piqueira. Weighting order and disorder on complexity measures, J. Taibah Univ. Sci. (2016), http://dx.doi.org/10.1016/j.jtusci.2016.05.003

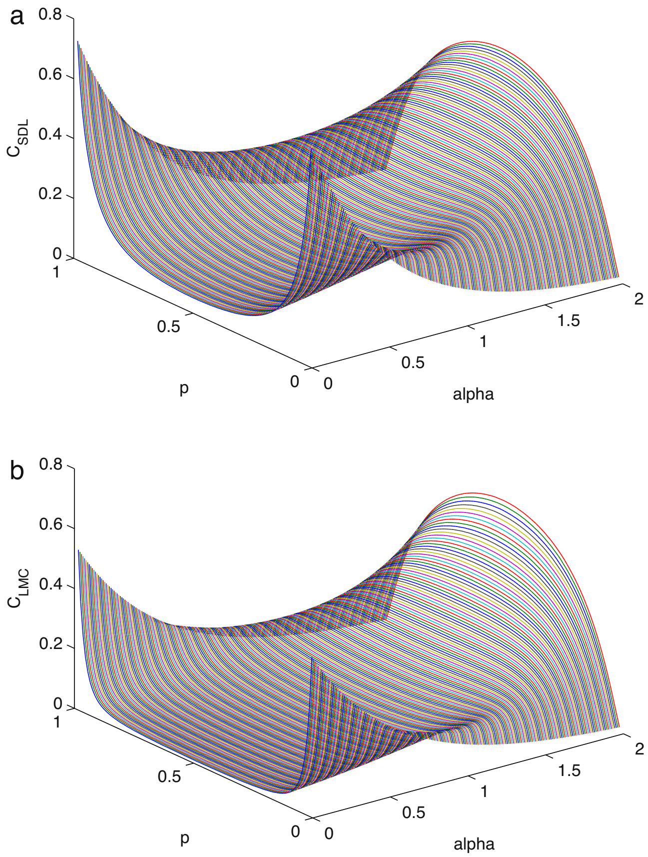

Fig. 5. SDL and LMC complexity measures depending on p and α ($N = 10$): (a) SDL and (b) LMC.

case, the complexity measures are minimum, but not zero.

Changing the importance of the terms, with $\alpha = 1.5$, the results are shown in Fig. 3a, for $N = 1$, and Fig. 3b, for $N = 10$. The symmetry related to equiprobability is maintained, with the maximum possible values of LMC and SDL measures slightly greater than the equiprobable case. When the number of trials increases, the minimum values of the LMC and SDL measures are not zero.

Increasing the order importance with $\alpha = 0.02$, the results are shown in Fig. 4a, for $N = 10$. The symmetry related to the equiprobable case is also maintained, but the aspect of LMC and SDL curves changes showing apparent monotonic decrease in the interval [0; 0.5] and increase in the interval [0.5; 1].

Changing the importance of the terms, with $\alpha = 1.98$, the results are shown in Fig. 4b, for $N = 10$. The symmetry related to the equiprobable case is maintained, but the shape curves of LMC and SDL measures drastically changes. The LMC and SDL measures become smooth functions of the individual success probability, with LMC and SDL measures practically superposed and follow the informational entropy function [16].

4. Conclusions

As shown in the former section, weighting the order (thermodynamical disequilibrium) and disorder (thermodynamical equilibrium) to calculate LMC and SDL complexity measures, provides the adaptation of the mathematical expressions to the explanation of the onset of complex behaviors in natural phenomena.

Concerning to the probability dependence of SDL complexity measure while changing the weights of the order and disorder terms, Fig. 5a confirms the symmetry related to the equiprobable case and how the shape of the curve changes, summarizing the conclusions for this measure.

For the probability dependence of LMC complexity measure while changing the weights of the order and disorder terms, Fig. 5b confirms the symmetry related to the equiprobable case and how the shape of the curve changes, summarizing the conclusions for this measure.

Finally, Fig. 5a and b expresses in a compact view that, qualitatively, LMC and SDL complexity measures are equivalent, even changing the weights.

References

[1] C. Petzold, The Annoted Turing, Wiley Publishing Inc., Indianapolis, Indiana, USA, 2008.
[2] A.N. Kolmogorov, Three approaches to the definition of the concept “quantity of information”, Probl. Pereda. Inf. 1 (1965) 3–11.
[3] E. Desurvive, Classical and Quantum Information Theory, Cambridge University Press, Cambridge, UK, 2009.
[4] C.E. Shannon, W. Weaver, The Mathematical Theory of Communication, Illini Books Edition, Urbana and Chicago, USA, 1963.
[5] H. Haken, Information and Self-Organization, Springer-Verlag, Berlin, Germany, 2000.
[6] G. Nicolis, I. Prigogine, Self-Organization in Nonequilibrium Systems, John Wiley & Sons, USA, 1977.
[7] M. Anand, L. Orlóci, Complexity in plant communities: the notion of quantification, J. Theor. Biol. 179 (1996) 179–186.
[8] K. Kaneko, I. Tsuda, Complex Systems: Chaos and Beyond, Springer Verlag, Berlin, Germany, 2001.
[9] R. López-Ruiz, H.L. Mancini, X. Calbet, A statistical measure of complexity, Phys. Lett. A 209 (1995) 321–326.
[10] J. Shiner, M. Davison, P. Landsberg, Simple measure for complexity, Phys. Rev. E 9 (2) (1999) 1459–1464.
[11] X. Portell, M. Ginovart, R. Carbó, J. Vives-Rego, Differences in stationary-phase cells of a commercial *Saccharomyces cerevisiae* wine yeast grown in aerobic and microaerophilic batch cultures assessed by electric particle analysis, light diffraction and flow cytometry, J. Ind. Microbiol. Biotechnol. 38 (2011) 141–151.
[12] J.R.C. Piqueira, S.H.V.L. de Mattos, J. Vasconcellos-Neto, Measuring complexity in three-trophic level systems, Ecol. Model. 220 (2009) 266–271.
[13] M. Paulescu, V. Badescu, Disorder and complexity measures for the stability of the daily solar radiative regime, UPB Sci. Bull. Ser. A 73 (3) (2011) 185–190.
[14] M. Mazza, M. Pinho, J.R.C. Piqueira, A.C. Roque, A dynamical model of fast cortical reorganization, J. Comput. Neurosci. 16 (2) (2004) 177–201.
[15] J.R.C. Piqueira, L.H.A. Monteiro, T.M.C. Magalhães, R.T. Ramos, R.B. Sassi, E.G. Cruz, Zipf’s law organizes a psychiatric ward, J. Theor. Biol. 198 (1999) 439–443.
[16] J.R.C. Piqueira, A comparison of LMC and SDL complexity measures on binomial distributions, Physica A 444 (2016) 271–275.
[17] R.B. Ash, Basic Probability Theory, Dover, USA, 2008.